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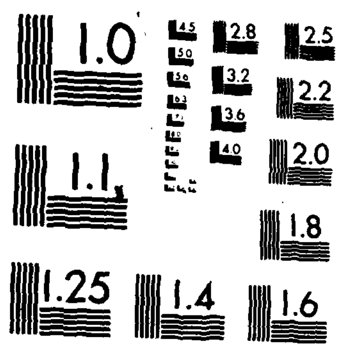
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Evolution of the transverse modes in a FEL, and application to the Orsay experiment

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Abstract

We derive the most general equations of motion for the electrons and the electromagnetic field in a free electron laser including the effects of diffraction and pulse propagation. The field evolution is expressed in terms of the amplitudes and phases of a complete set of transverse modes. The analytic solution is given in the small signal regime, where the theory is shown to be in excellent agreement with a recent experiment at Orsay.

Introduction

In this paper, we summarize a new approach[1] to calculating the three dimensional effects operative in free electron lasers. The previously mentioned approaches consider the growth of the field $\mathcal{E}(r,t)$ along the propagation or z axis by evaluating its change at each point (x,y), and integrating numerically through the interaction region in the time domain[2-7] or in the frequency domain.[8] These techniques all demand long computer runs if they are to be applied to a real experimental situation. Our approach decomposes the problem into the minimum number of physically observable quantities: the transverse optical modes of the system. The field evolution is expressed in terms of a complete set of orthogonal transverse modes; equations are developed for the propagation of the amplitude and phase of each mode. In physical systems which operate on a few of the lowest order modes, this approach greatly increases the accuracy, and may reduce the required computer time for the calculation by working in a vector space well matched to the solution of the problem. For the oscillator case, the appropriate choice of modes is the set of eigenmodes of the cavity. For the amplifier, the vector space of modes is determined by the characteristics of the input mode, which is presumably a TEM_{00} Gaussian mode. In either device, an optimum design would result in the excitation of as few of the higher order modes as possible. The modal decomposition method is therefore well adapted to the prediction and optimization of the operation of the free electron laser.

We begin by developing the most general equations for the propagation of the mode amplitudes and phases in a FEL. These equations are then specialized to the small signal case, and applied to an experiment we have recently performed at Orsay.[9]

1. Theoretical development of the fundamental equations

The FEL system is properly described by the coupled Maxwell and Lorentz force equations. From these, we shall derive a self-consistent set of equations describing the electron and the transverse optical mode dynamics. We use the dimensionless notation originally developed by W. B. Colson[10] (in fact this work is a generalization of Colson's work to include transverse modes and we shall stay as close as possible to his original notation).

We begin by re-deriving the equations for the slowly varying amplitude and phase of the electromagnetic field, but now including rigorously the transverse dependence. The field is described in free space by the paraxial wave equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-2ik\frac{\partial}{\partial z}\right)E(\vec{r},t)e^{i\phi(\vec{r},t)}=0 \qquad (1)$$

This equation is derived from the wave equation $\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathcal{E}=0$ in the slowly varying amplitude and phase approximation. The general solution of Eq. (1) can be expressed as a linear combination of a complete set of orthogonal modes. If we define these modes by the complex amplitude $E_m e^{i\phi_m}$ where E_m is real and m is the generalized index of the mode (in the two dimensional transverse space we consider, m represents two integer numbers). The orthogonality relation reads

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$$\int \frac{dxdy}{\pi w_o^2} E_m e^{i\psi_m} E_n e^{-i\psi_n} = \delta_{mn} \tag{2}$$

where we have chosen a convenient normalization which makes the E_m dimensionless. Although we will use the cylindrical modes in the examples, we proceed with the general theoretical development which makes no assumptions on the specific form of the modes.

In the FEL, the coefficients c_m in the mode expansion become time dependent. We wish to calculate the evolution of the amplitude and phase of these mode coefficients.

$$\frac{\partial \nu}{\partial \tau} = \sum_m |a_m| E_m \cos(\zeta + \psi_m + \phi_m) \tag{3}$$

$$\frac{\partial \zeta}{\partial \tau} = \nu \tag{4}$$

$$\frac{\partial a_m}{\partial \tau} = - \int \frac{dxdy}{\pi w_o^2} r E_m e^{-i\psi_m} \langle e^{-i\zeta} \rangle_{\zeta_o \nu_o} \tag{5}$$

where we have made the definitions

$$a_m(z,t) \equiv \frac{4\pi eNLK}{\gamma^2 mc^2} c_m(z,t) \tag{6}$$

$$r(\vec{r},t) \equiv \frac{8\pi^2 e^2 N L^2 K^2}{\gamma^3 mc^2} \rho(\vec{r},t) \tag{7}$$

$$c_m(z,t) = |c_m(z,t)| \, e^{i\phi_m(z,t)} \tag{8}$$

and where

$$\zeta(t) = (k + k_o)z(t) - \omega t \qquad \text{dimensionless electron phase} \tag{9}$$

$$\nu(t) = L\left[(k + k_o)\beta_z(t) - k\right] \qquad \text{dimensionless resonance parameter} \tag{10}$$

$$\tau = \frac{ct}{L} \qquad \text{dimensionless interaction time} \tag{11}$$

Here we consider an N period helical undulator of length L, magnetic period $\lambda_o = 2\pi/k_o$, peak magnetic field B, and deflection parameter $K = 93.4\ B(\text{gauss})\lambda_o(\text{cm})$. An electron beam of energy γmc^2, and number density ρ travels along the axis of the undulator; an individual electron has longitudinal coordinate $z(t)$ and longitudinal velocity $c\beta_z(t)$ at time t. An arbitrary helically polarized wave of wavelength $\lambda = 2\pi/k$, frequency ω, and electric field $\mathcal{E}(z,t) = E(z,t)e^{i(kz-\omega t+\phi(z,t))}$ interacts with the electrons, and c_m are the mode coefficients in the expansion. In Eq. (5), $\langle\rangle_{\zeta_o \nu_o}$ is the average over the initial phase ζ_o and resonance parameter ν_o of the electron population at the position z.

Eqs. (3) and (4) are derived directly from the Lorentz force equation and describe the effect of the radiation field on the electrons. The work done by the longitudinal field on the electrons is neglected here, which is a good approximation provided that the modes are not too divergent[7] $\lambda/w_o \ll 2\pi^2 KN/\gamma$. Eq.(5) is derived from the Maxwell equations and describes the effect of the electron on the radiation field. The set (3)-(5) is self-consistent. Indeed those equations are very close to being the most general classical equations describing the FEL dynamics. They apply to high and low gain devices ($r' \gg 1$ or $r' \ll 1$), high field and low field cases ($a' \gg 1$ or $a' \ll 1$), and include the effects of multiple longitudinal modes (laser lethargy effects) through the $\hat{z}$ dependence of r', E, and ϕ. Slight modifications allow their extension to the cases of a) the planar undulator,[1,11] b) the tapered undulator,[10] c) the optical klystron,[7] and d) space charge effects.[11] For simplicity in the following development, we drop the explicit $\hat{z}$ dependence which has been thoroughly discussed by Colson,[10] and concentrate on the transverse phenomena.

Equation (5) describes the growth or decay of the radiation field due to its interaction with the electrons. It shows clearly the fact that the growth in the m^{th} mode amplitude and

phase is given by the overlap integral of the in-phase and out-of-phase components of the charge density with the complex conjugate of that mode, as one would expect. We note that the only assumptions made on the modes $E_m e^{i\psi_m}$ used in (3)-(5) are orthogonality and completeness. This means these equations are also valid for the cases of waveguide modes and dielectrically loaded cavities.

The Eqs. (3)-(5) can be integrated numerically to find the evolution of the optical wave in any Compton regime FEL. In a high field experiment, Eqs. (4) and (5) are nonlinear in a, and the wave evolution can only be obtained numerically. In this case, (3)-(5) provide a precise and efficient technique for solving the general problem. In a low field situation such as we find at Orsay, however, the problem becomes linear, and can be solved analytically. We proceed with the low field case in the next section.

2. The low fields solution

The low field case is defined by $|a_m| \ll 1$ for every mode. In other words, the electrons do not become overbunched. Experiments which operate in this domain include the low field amplifier experiments, and storage ring FEL oscillators which saturate by mechanisms other than overbunching. The ignition of any FEL oscillator also occurs in this domain.

Equations (3)-(5) can be solved by integrating (3) and (4) to lowest order in the fields a_m and inserting the result for ζ into Eq. (5). If the electrons are uniformly distributed initially in phase, we find

$$\frac{\partial a_m(\tau)}{\partial \tau} = \int_0^{\tau} d\tau' \int_0^{\tau'} d\tau'' \sum_n M_{mn}(\tau,\tau'')a_n(\tau'') \tag{12}$$

where

$$M_{mn}(\tau,\tau'') = \frac{i}{2}\int\frac{dx\,dy}{\pi w_0^2}\, r(x,y)E_m(x,y,\tau)E_n(x,y,\tau'') \cdot e^{-i\left((\psi_m(x,y,\tau)-\psi_n(x,y,\tau'')\right)} \left\langle e^{-i\nu_0(\tau-\tau'')}\right\rangle_{\nu_0} \tag{13}$$

Equation (12) describes a linear evolution of the mode amplitudes, and upon integration, gives the relation

$$a_m(\tau = 1) = \sum_n (I + G)_{mn} a_n(\tau = 0) \tag{14}$$

where I is the identity matrix, and G, which is generally not Hermitian, has elements

$$g_{mn} = \int_0^1 d\tau \int_0^{\tau} d\tau' \int_0^{\tau'} d\tau'' M_{mn}(\tau,\tau'') + \int_0^1 d\tau_1 \int_0^{\tau_1} d\tau_2 \int_0^{\tau_2} d\tau_3 M_{m\ell}(\tau_1,\tau_3) \int_0^{\tau_3} d\tau_4 \int_0^{\tau_4} d\tau_5 \int_0^{\tau_5} d\tau_6 M_{\ell n}(\tau_3,\tau_6) + \ldots \tag{15}$$

The higher order terms in g_{mn} are proportional to r^2 and higher powers of r, and are negligible in the low gain case.

Evidently this matrix is of great interest since multiple passes of the electron beam will result in multiple products of this matrix, greatly simplifying the calculation of the modes' growth. We shall discuss the consequences for an oscillation experiment in Sect. 2.2.

Of course, one must keep in mind that energy is radiated into other modes, and that cross terms will mix a multiple mode input. If the input beam is truly monomode, the power radiated into the n^{th} mode is lower than that into the m^{th} mode by the ratio $|g_{mn}|^2/2\mathrm{Re}(g_{mm})$ which is small for low gain ($r \ll 1$) systems. It is only in this case that it makes sense to speak of the gain of a mode. In high gain systems, however, the off-diagonal terms can lead to substantial emission of energy into the higher order transverse modes. If the input beam is multimode, of course, mode mixing occurs at all power levels.

Let us calculate g_{mn} in the simple case of experimental interest where the electron beam is cylindrical, and a good choice of modes is the cylindrical cavity eigenmodes. We restrict ourselves to the weakly diverging case $\pi w_o^2 \gg \lambda L$ where the gain takes on its most familiar form. The mode amplitudes and phases in (13) become independent of τ, and we can integrate the first term in (15) to find the gain. For small spread in resonance parameter over the beam phase space, the integral gives the well known gain spectrum

$$g_{mn} = \int \frac{dxdy}{\pi w_o^2} r(x,y) E_m(x,y) E_n(x,y) e^{-i\psi_m(x,y)} e^{i\psi_n(x,y)}$$

$$\cdot \left\{ \frac{1 - \cos\nu_c - \frac{\nu_c}{2}\sin\nu_c}{\nu_c^3} + i \frac{-\frac{\nu_c}{2} - \frac{\nu_c}{2}\cos\nu_c + \sin\nu_c}{\nu_c^3} \right\} \tag{16}$$

In the usual experimental case (unfortunately), $|g_{mn}| \ll 1$ and the energy gain G_m on the mode m becomes

$$G_m = 2\mathrm{Re}(g_{mm}) = \int \frac{dxdy}{\pi w_o^2} 2rE_m^2 \left(\frac{1 - \cos\nu_c - \frac{\nu_c}{2}\sin\nu_c}{\nu_c^3} \right) \tag{17}$$

This is exactly the gain one calculates by using the filling factor obtained by integrating the mode profile overlap with the gain profile. Specializing to the TEM_{oo} case with a Gaussian electron beam of width σ, we find

$$G_{oo} = 2r_o \left(\frac{1 - \cos\nu_c - \frac{\nu_c}{2}\sin\nu_c}{\nu_c^3} \quad \frac{1}{1 - \frac{w_o^2}{4\sigma^2}} \right) \tag{18}$$

complete with the familiar filling factor.

The ν_c dependence of G_m is the well known spectral dependence. The imaginary part of g_{mn} is not new. It describes the phase shift of the radiation field as described by Colson.[10] The effect of the divergence of the beam on the diagonal terms in G is, <u>to first order</u>, and for a filamentary electron beam, the addition of a time-varying phase which shifts the resonance curve in Eq. (18) by a constant depending on the mode

$$\nu_c \to \nu_c - \frac{\lambda L}{\pi w_o^2} (2p + \ell + 1) \tag{19}$$

This means that the gain curves of the modes are shifted with respect to each other. This phenomenon has been calculated for the fundamental TEM_{oo} mode in the energy loss approximation,[14] and has recently been observed experimentally at Orsay.[16] It should be noted that for many practical situations where the cavity is optimized for gain on the TEM_{oo} mode, this expression is valid for only the lowest order mode. The higher modes become distorted in form as well as simply shifted in resonance parameter by equation (19).

In another paper, we discuss the optimization of the cavity in a low field oscillator.[1]

3. Application to gain-vs-aperture experiment

3.1 Description of the experiment

The gain of the Orsay FEL has recently been measured with the optical klystron in place[12] in an amplifier experiment using an external argon ion laser to provide the coherent mode. A detailed description of the apparatus can be found elsewhere.[13] The laser beam is analyzed at a distance d from the optical klystron after passing through an adjustable collimating iris, which is centered on the laser mode emerging from the interaction region. The gain is measured as the ratio of the power detected in phase with both the electron repetition frequency and the chopper frequency (the amplified power) divided by the power in phase with the chopper alone (the incident laser power). Calibration is performed as in our previous work.[13]

The gain is recorded as a function of the iris aperture, and large variations are observed.[9] One set of data points is reproduced in Fig. 1, where the gain is normalized to its value for the iris completely open, and the iris diameter is normalized to the measured beam waist at the iris. The data is taken at maximum gain, which means $v_o \approx 0$ for the optical klystron, and the laser beam was carefully aligned to within about .05 mm of the axis of the electron beam. The change in the measured gain as the iris is closed means that the laser is not uniformly amplified in its transverse profile. In fact, this experiment provides a very sensitive technique for measuring the power emitted into the higher order modes even in the small gain limit and for a monomode input beam. Clearly a calculation of the g_{mn} is necessary in order to explain these results. In the next section, we apply the theory we have developed to the case at hand, and in Sect. 3.3, precise comparison is made between the experimental and the theoretical results.

3.2 Multimode emission in a single mode amplifier experiment

In this section, we assume the incident wave is a single mode TEM_{00} beam with a weak field ($|a_o| < 1$), and perfectly aligned onto the electron beam. As discussed previously, we take the cylindrical eigenmodes based on the form of the input laser beam. Using the notation of Sect. 1, the input laser field reads

$$E^I(\vec{r}) = c_o E_o(\vec{r}) e^{i\psi_o(\vec{r})} \tag{20}$$

where the subscript 0 refers to the TEM_{00} mode. From (14), the output field $E^S(\vec{r})$ becomes

$$E^S = E^I + c_o \sum_{j=0}^{\infty} g_{jo} E_j(\vec{r}) e^{i\psi_j(\vec{r})} \tag{21}$$

Assuming low gain, the output power passing through the iris aperture is

$$\frac{8\pi P}{c} = \int dS\,|E^S|^2 \approx c_o^2 \int dS E_o^2 + 2c_o^2 \mathrm{Re}\left\{\sum_{j=0}^{\infty} g_{jo} \int dS E_o E_j^* e^{i(\psi_o - \psi_j)}\right\} \tag{22}$$

where $\int dS$ covers the iris aperture. The gain is therefore

$$G = \frac{2\mathrm{Re}\left\{\sum_{j=0}^{\infty} g_{jo} \int dS E_o E_j^* e^{i(\psi_o-\psi_j)}\right\}}{\int dS E_o^2} \tag{23}$$

For purely cylindrical $\ell = 0$ modes, G can be written

$$G = 2\mathrm{Re}\left\{g_{oo} + \sum_{p=1}^{\infty} g_{p_o} e^{i2p\tan^{-1}\frac{z}{z_o}} \frac{\int_0^X L_p(x)e^{-x}dx}{\int_0^X e^{-x}dx}\right\} \tag{24}$$

where z_o is the Rayleigh range of the laser mode, z is the distance between the iris and the laser beam waist, and $X = r_o^2/2w^2(z)$ where r_o is the iris diameter. There are two interesting limiting cases:

$$G = 2\mathrm{Re}(g_{oo}) \qquad X \to \infty \text{ (iris open)} \tag{25}$$

$$G = 2\mathrm{Re}\left[(g_{oo} + \sum_{p=1}^{\infty} g_{po} e^{i2p\tan^{-1}\frac{z}{z_o}}\right] \qquad X \to 0 \text{ (iris closed)} \tag{26}$$

It is obvious from Eqs. (24)-(26) that the gain changes with iris diameter in a way which depends on the magnitudes of the off-diagonal terms in the gain matrix.

The generalization is straightforward to the case of the multimode input beam, and to imperfect alignment of the laser and electron beams, although the calculation becomes more difficult. This calculation also applies to high power input laser beams ($a_o >> 1$) provided one keeps in mind that the g_{po} are functions of a_c.

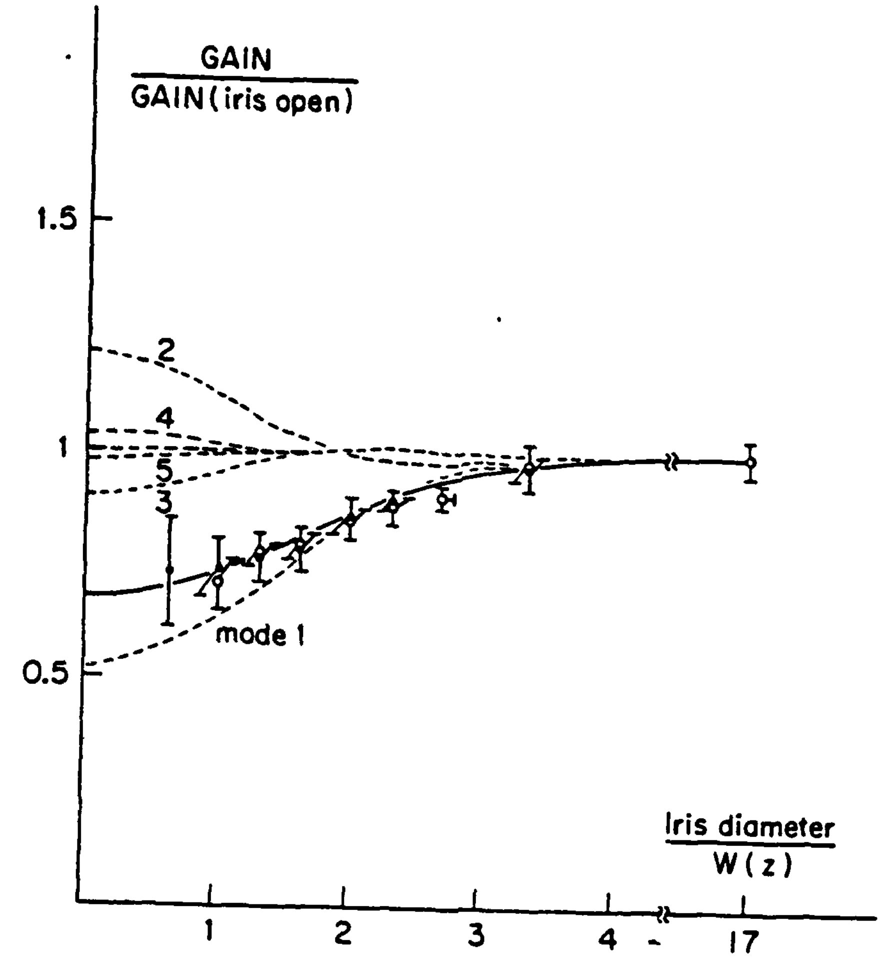

Figure 1. The measured gain as a function of the iris diameter[9] normalized to the measured beam waist at the iris. The solid points were taken closing the iris and the open points while opening it. The error bars are the one sigma statistical errors. All points have the same horizontal error bar which is shown for the point at 2.7. The solid curve is calculated using the measured values for the electron and laser beam sizes. The effect of each higher order mode is shown by the dashed curves.

3.3 Application to the Orsay experiment

The experimental points shown on Fig. 1 were taken under the following approximate conditions

laser beam:
- measured beam waist w_o = .67 mm
- wavelength = 5145 Å
- measured beam waist at iris $w(z)$ = 2.7 mm
- distance from optical klystron to iris d = 11.6 m

electron beam:
- gaussian and cylindrical with σ = .32 mm

optical klystron:
- Nd = 80[11,12,14]
- resonance parameter corresponding to maximum gain with iris open

The dark curve has been calculated using Eqs. (12), (13) and (24) for the planar configuration[1,11] taking into account the 10 lowest order $\ell = 0$ modes. The dashed curves show the contribution of each individual mode. The agreement between the experimental results and the theory is excellent.

Very similar curves were calculated for other resonance parameters indicating that as expected, the diffraction effects do not change much as a function of detuning parameter for modes with low divergence. The effects of the transverse size of the electron beam and the distance of the analyzing aperture from the FEL are discussed elsewhere.[1,5]

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